4226. *Proposed by Daniel Sitaru.*

Prove that if $0 < a < b$ then:

$$\Big(\int_a^b \frac{\sqrt{1+x^2}}{x}dx\Big)^2 > (b-a)^2 + \ln^2\Big(\frac{b}{a}\Big).$$

We received nine submissions, eight of which are correct and the other is incorrect. We present a composite of virtually the same solutions by Arkady Alt; Michel Bataille; M. Bello, M. Benito, Ó. Ciaurri, E. Fernández, and L. Roncal (jointly); and Digby Smith.

Note first that

$$\begin{aligned}
&\left(\int_a^b \frac{\sqrt{1+x^2}}{x}dx\right)^2 > (b-a)^2 + \ln^2\frac{b}{a}\\
&\iff \left(\int_a^b \frac{\sqrt{1+x^2}}{x}dx\right)^2 - \left(\int_a^b \frac{1}{x}dx\right)^2 > (b-a)^2\\
&\iff \int_a^b \frac{\sqrt{1+x^2}+1}{x}dx \cdot \int_a^b \frac{\sqrt{1+x^2}-1}{x}dx > (b-a)^2. \qquad (1)
\end{aligned}$$

Let $f(x) = \dfrac{\sqrt{1+x^2}+1}{x}$, $x \in [a,b]$. Then $f(x) > 0$ and $\dfrac{1}{f(x)} = \dfrac{\sqrt{1+x^2}-1}{x}$. By the integral form of the Cauchy-Schwarz Inequality, we have

$$\begin{aligned}
\left(\int_a^b f(x)\,dx\right)\left(\int_a^b \frac{1}{f(x)}dx\right) &= \left(\int_a^b \left(\sqrt{f(x)}\right)^2 dx\right)\left(\int_a^b \left(\sqrt{\frac{1}{f(x)}}\right)^2 dx\right)\\
&\geq \left(\int_a^b 1dx\right)^2\\
&= (b-a)^2. \qquad (2)
\end{aligned}$$

But equality cannot hold in (2) as f is not a constant on $[a,b]$. Hence, from (1) and (2) the result follows.

4227. *Proposed by Dan Marinescu and Leonard Giugiuc.*

Let P be a point in the interior of an equilateral triangle ABC whose sides have length 1, and let R' and r' be the circumradius and inradius of the triangle whose sides are congruent to PA, PB and PC (which exists by Pompeiu's theorem). Prove that

$$3R' \geq 1 \geq 6r'.$$

Among the four submissions, three were complete and correct; in the fourth, Michel Bataille simply provided a reference where the proof can be found: Proposition 7 in József Sándor's "On the Geometry of Equilateral Triangles", Forum Geometricorum, *vol. 5 (2005) 107-117. Here we present the solution by Roy Barbara.*